Long-Term Monitoring Strategy using Nonlinear Kalman Filtering

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Present State

- Existing computer models are limited
- Monitoring provides assurance
- Model revision rationale is needed

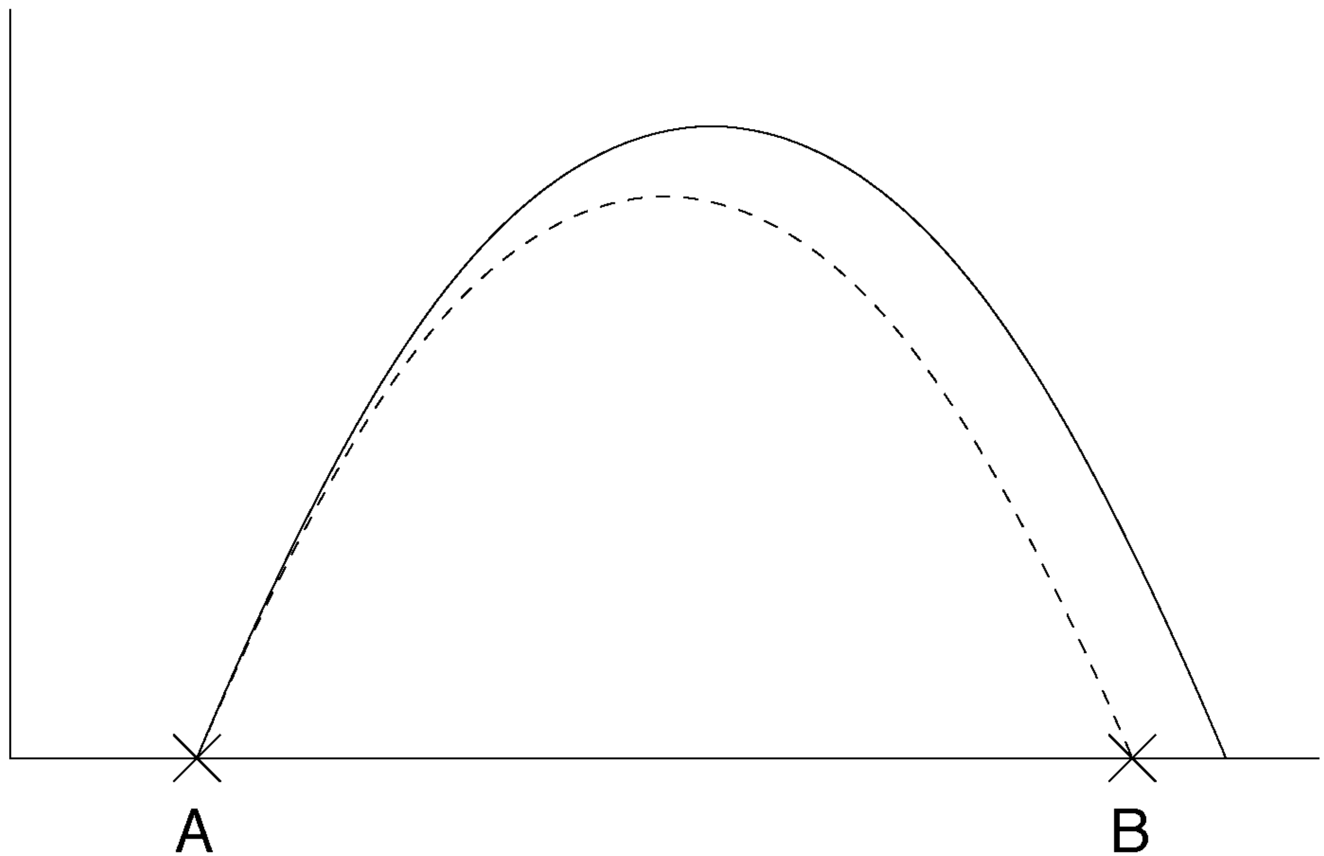
Objectives

- Long-Term Monitoring
- Rational Monitoring Strategy
- Combine Models and Measurements

Classical Example

Rocket from A to B

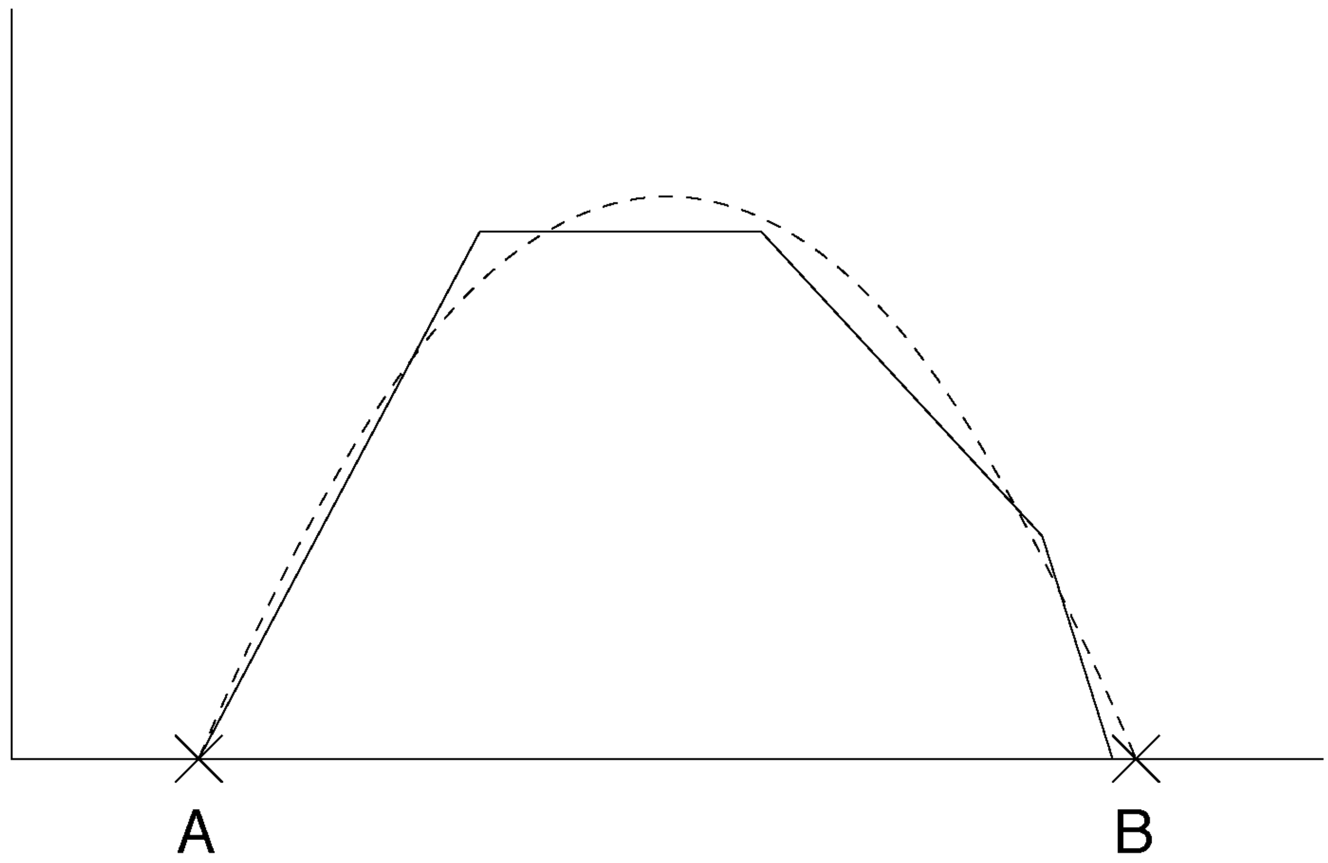
Accelerometer



Classical Example

Rocket from A to B

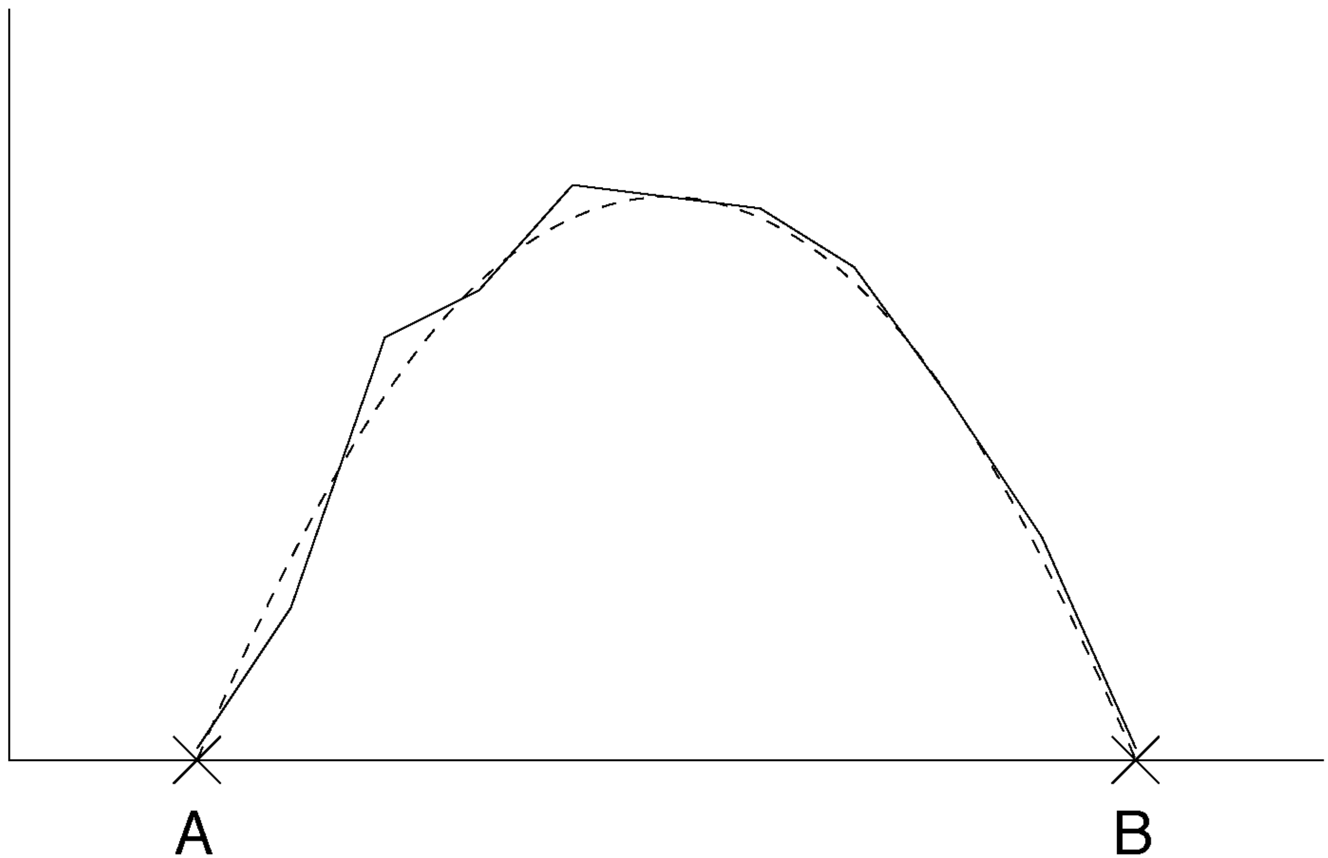
GPS positioning



Classical Example

Rocket from A to B

Accelerometer + GPS + Filtering



Kalman Filtering

- Process model: F = ma
- State vector: x, v, a
- Measurements: GPS
- Minimum state vector uncertainty

Kalman Filtering

General Description

State Vector: $$\mathbf{x} = \{x_0, x_1, \ldots, x_N\}$$

Propagation Model: $$\mathbf{x}(t + \Delta t) = \mathbf{f}[\mathbf{x}(t)]$$

Measurements: $$\mathbf{y} = \{y_0, y_1, \ldots, y_M\}$$

Predictions: $$\hat{\mathbf{y}}(t + \Delta t) = \mathbf{g}[\mathbf{x}(t + \Delta t)]$$

Kalman Filtering

Linear Filtering

Propagation Model:

$$\hat{\mathbf{x}}^{k-} = \mathsf{A}_k \, \hat{\mathbf{x}}^{k-1} + \mathsf{B}_k \, \mathbf{u}^{k-1}$$

Measurement Prediction:

$$\hat{\mathbf{y}}^{k} = \mathsf{C}_k \, \hat{\mathbf{x}}^{k-}$$

Kalman Filtering

Linear Filtering

Error Covariance Matrix:

$$\mathsf{P}_{k_-} = \mathsf{A}_k \mathsf{P}_{k-1} \mathsf{A}_k^{\mathrm{T}} + \mathsf{Q}_k$$

Kalman Gain Matrix:

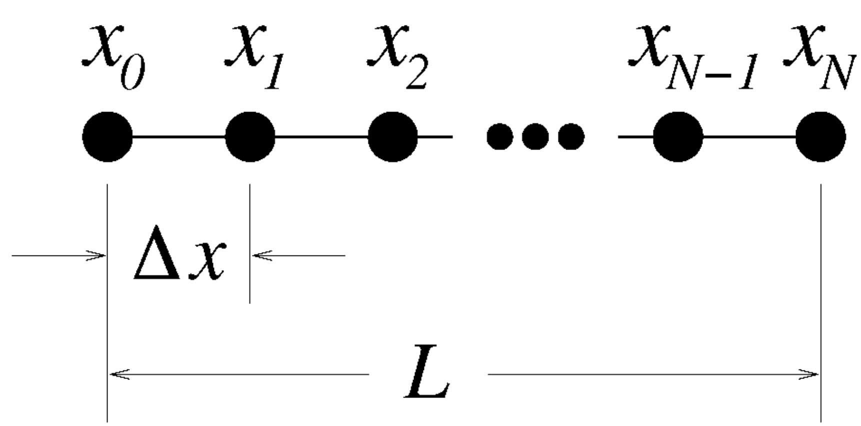
$$\mathsf{K}_k = \mathsf{P}_{k_-} \mathsf{C}^{\mathrm{T}} \left(\mathsf{C}_k \mathsf{P}_{k_-} \mathsf{C}^{\mathrm{T}} + \mathsf{R}_k \right)^{-1}$$

Filtered State Vector:

$$\hat{\mathbf{x}}^k = \hat{\mathbf{x}}^{k-} + \mathsf{K}_k \left[\mathbf{y}^k - \mathsf{C}_k \hat{\mathbf{x}}^{k-} \right]$$

Fickian Diffusion

$$\frac{\partial c}{\partial t} = D \nabla^2 c$$



$$\hat{\mathbf{x}}^{k-} = \mathsf{A}(\eta)\,\hat{\mathbf{x}}^{k-1}$$

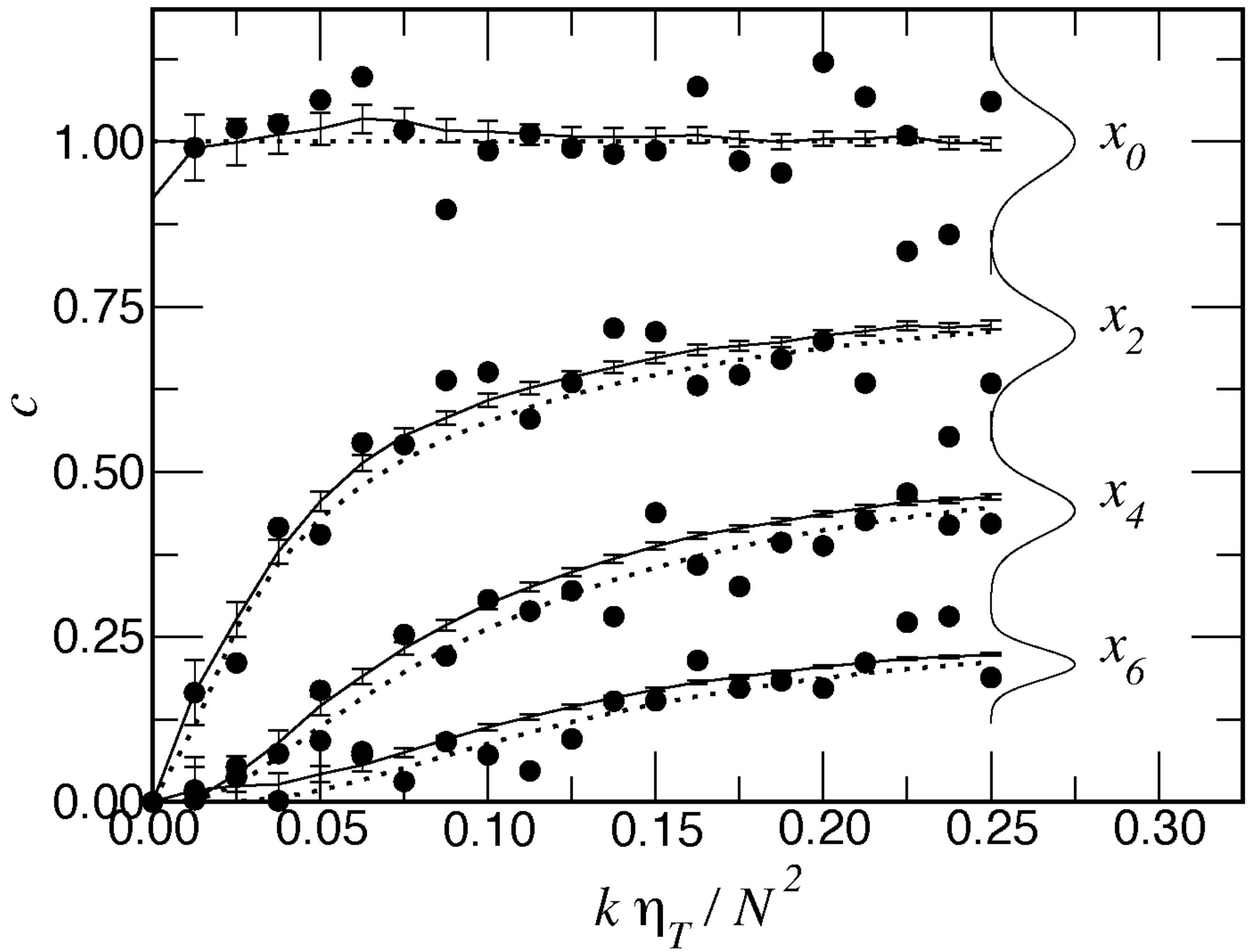
$$\eta = \frac{D\,\Delta t}{(\Delta x)^2}$$

$$c(0,t) = 1.00 \pm 0.05$$

$$\eta = 0.80 \pm 0.16$$

Fickian Diffusion

$$\eta = 0.9867 \qquad x_0 = 0.9140$$



Kalman Filtering

Extended Kalman Filtering (EKF)

Process Model:

$$\mathbf{x} = \left[x_0, x_1, \ldots, x_N, \eta\right]^{\mathrm{T}} \qquad \hat{\mathbf{x}}^{k-} = \mathbf{f}\left[\hat{\mathbf{x}}^{k-1}\right]$$

Jacobian Matrix:

$$\mathbb{F} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{bmatrix} \mathsf{A} & \mathbf{a}_\eta \\ \mathbf{0}^{\mathrm{T}} & 1 \end{bmatrix} \qquad (\mathbf{a}_\eta)_i = \frac{\partial f_i}{\partial \eta}$$

Kalman Filtering

Extended Kalman Filtering (EKF)

Error Covariance Matrix:

$$\mathsf{P}_{k_-} = \mathbb{F}_k \, \mathsf{P}_{k-1} \, \mathbb{F}_k^{\mathrm{T}} + \mathsf{Q}_k$$

Kalman Gain Matrix:

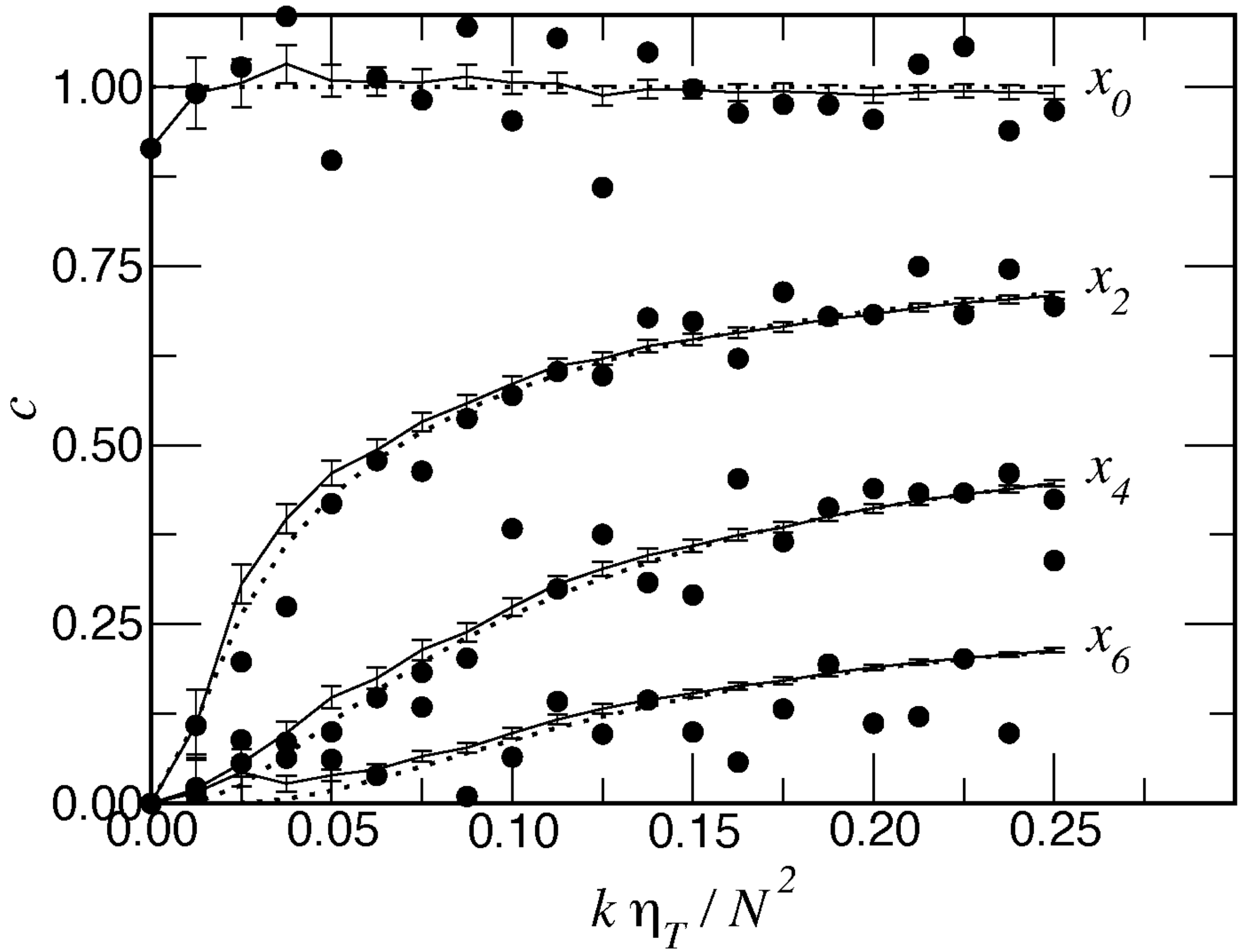
$$\mathsf{K}_k = \mathsf{P}_{k_-} \mathsf{C}^{\mathrm{T}} \left(\mathsf{C}_k \mathsf{P}_{k_-} \mathsf{C}^{\mathrm{T}} + \mathsf{R}_k \right)^{-1}$$

Filtered State Vector:

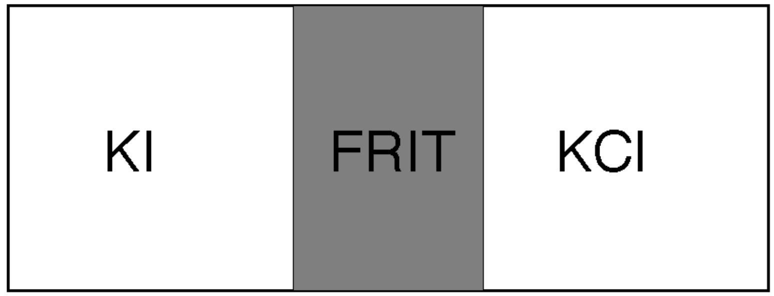
$$\hat{\mathbf{x}}^k = \hat{\mathbf{x}}^{k-} + \mathsf{K}_k \left[\mathbf{y}^k - \mathsf{C}_k \, \hat{\mathbf{x}}^{k-} \right]$$

Fickian Diffusion

Extended Kalman Filtering (EKF)



Diffusion Example



$$\mathbf{j}_i^e = -D_i \left(1 + \frac{\partial \ln y_i}{\partial \ln c_i}\right) \nabla c_i - z_i c_i u_i \nabla \psi$$

$$\frac{\partial \theta c_i}{\partial t} = -\nabla \cdot \left(\frac{\mathbf{j}_i^e}{\Upsilon}\right)$$

$$F \sum_i z_i \mathbf{j}_i = 0$$

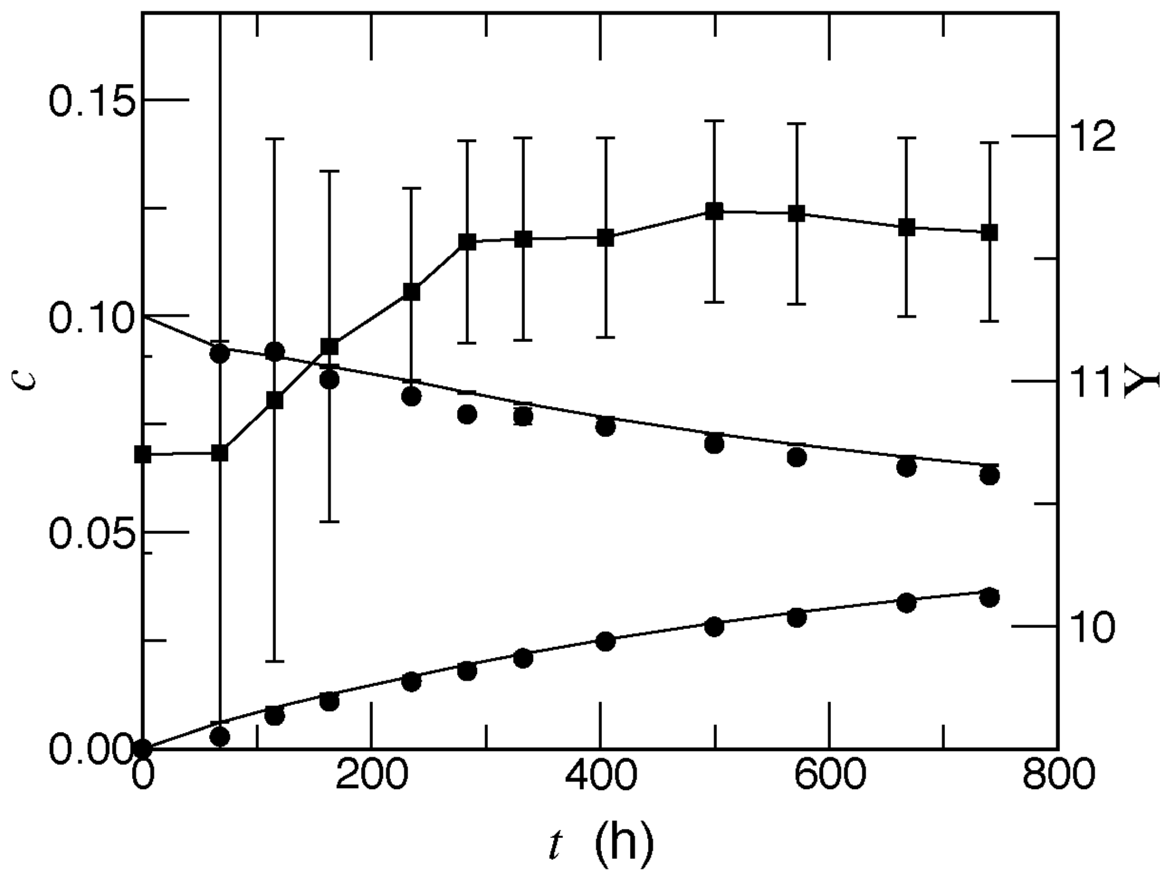
Diffusion Example

State Vector:

$$\mathbf{x} = \begin{bmatrix} \mathbf{I} \\ \mathbf{Cl} \\ \theta \\ \Upsilon \end{bmatrix}$$

Diffusion Data

KI/KCl



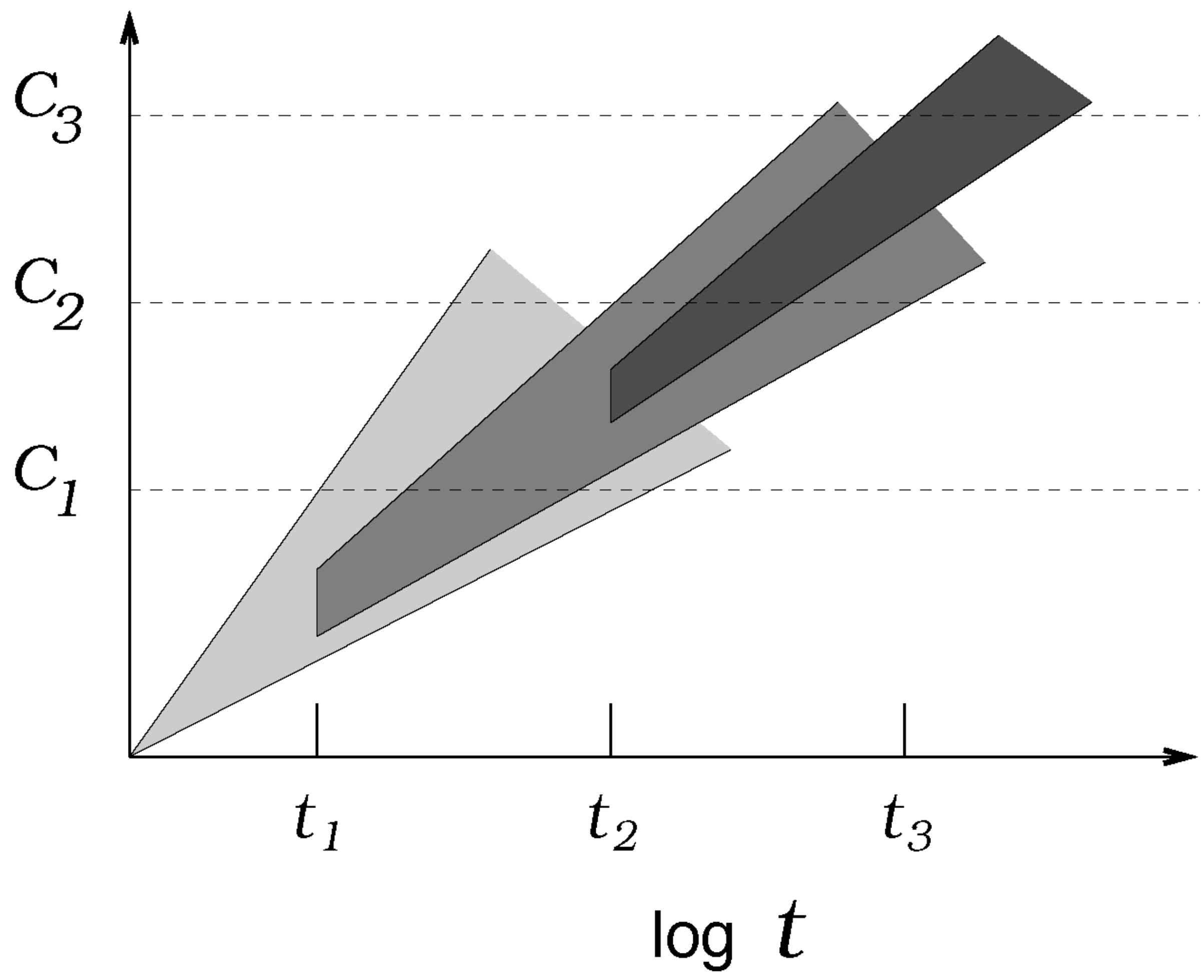
Snyder, Lu, and Philip, *Eur. J. Phys.*, (to be published) 2007.

Future Directions

- Other nonlinear Kalman filters
- Optimum state vector basis set
- State vector constraints

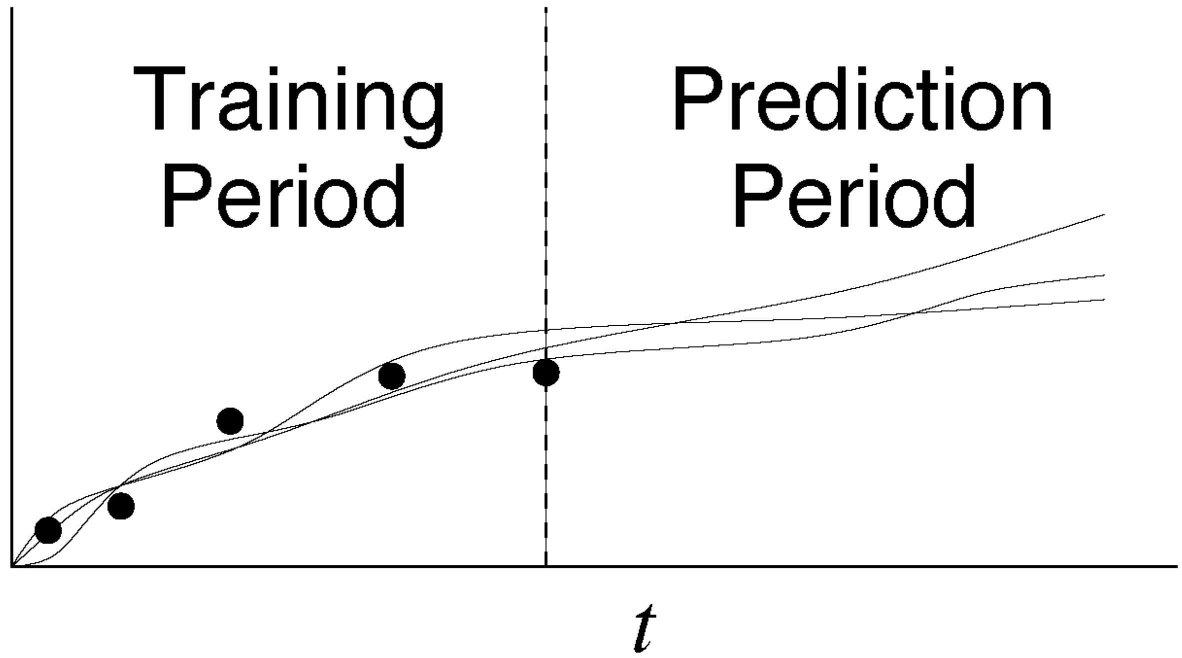
Monitoring Analogy

Measurement Thresholds



ML-BMA

Maximum Likelihood - Bayesian Model Averaging



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Summary

- Rational basis for long-term monitoring
- Initial measurements “prime” the filter
- Utility of surrogate samples
- Analysis of laboratory data